Magnetic Field (E5)

Objectives

- Measure the magnetic fields produced by coils.
- Measure both horizontal and vertical components of the Earth's magnetic field.
- Know how to use the 'Right hand' rule.

Theory

A. Magnetic Fields

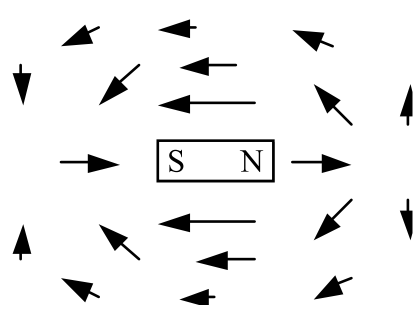
Magnetic interactions between magnets or between moving charges are through magnetic fields. Magnetic field lines that represent the magnetic field always form closed loops. The end of the magnet from which the field lines emerge is defined as the north pole of the magnet, while the end to which the field lines enter is called the south pole. The number of magnetic field lines per unit volume represents the magnitude of the magnetic field.

The SI unit of magnetic field is called *Tesla* (T) and is equivalent to:

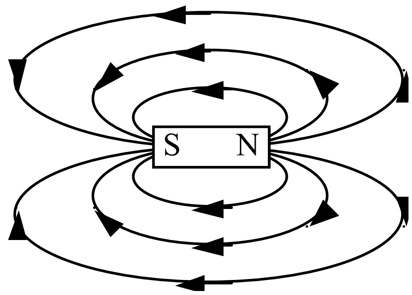
$$1\ \mathrm{T} = 1\ \frac{\mathrm{N}}{\mathrm{A\ m}} = 1\ \frac{\mathrm{N\ s}}{\mathrm{C\ m}}$$

The old cgs unit of magnetic field strength is called *Gauss* (Gs). The conversion factor from Gauss to Tesla is equal to $1\ \mathrm{T} = 10\ 000\ \mathrm{Gs}$.

A **magnetic field** is described by a vector field; that is, at every point in space, a three-dimensional vector describes the direction and the magnitude of the field. The magnetic field vector (at any point) has the same direction as a small compass needle placed at that point. This is illustrated in the following diagram.



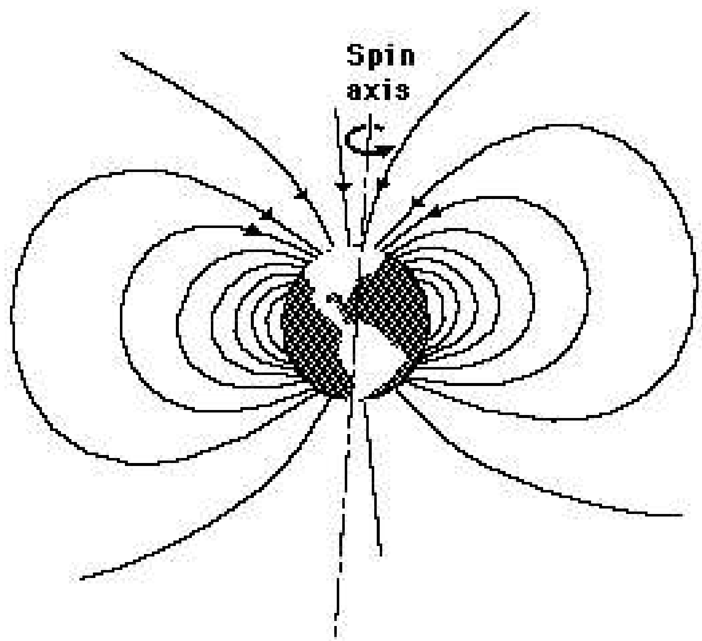
However, vector field diagrams like these are sometimes hard to visualize. Therefore, we draw them using field lines.



Here, you see the magnetic field lines with arrowheads pointing from the north pole to the south pole. These lines are not real; they are only a graphical representation of the points in space, which all have the same magnetic field strength. At any point on a field line, the magnetic field vector is pointing tangent to the field line.

At any point in space, all the magnetic field vectors present from all magnetic sources at that point will combine to make the net field vector. This vector sum is called the **superposition** of all the vector components; this principle is called the **superposition principle**.

B. Earth's Magnetic Fields



The Earth has its own magnetic field. This fact may be used to determine the directions of magnetic field lines for the Earth or any other magnet. The north pole of a bar magnet (or any other magnet for that matter) will tend to face the earth's geographical north pole. Likewise, a magnet's south pole will tend to face the Earth's geographical south pole. Since opposites attract, it is clear that the geographical North Pole is actually the magnetic south pole!

Please, note that we are located much closer to the geographical North Pole than to the geographical South Pole. Therefore, we should expect a nonzero vertical component of the Earth's magnetic field. You should also know that the location of the magnetic poles does change with time and it does not coincide with geographical poles (i.e., places, where axis of rotation intersects the Earth's surface).

Therefore, knowing this, you can use a compass as a probe for finding the direction of a magnetic field at a point in space. The direction of a magnetic field line at any point is taken as the direction that the north pole of a compass needle points to at this location. This is assuming, of course, that a compass needle is free to point in all three dimensions. If the needle is bound to move only in two dimensions (as in a standard magnetic compass) then we will need to make two independent measurements to get the horizontal and vertical components of the Earth's magnetic field.

C. Helmholtz Coils

Two thin coaxial coils, each containing the same number of turns and carrying the same current, can be used to obtain a nearly uniform value of magnetic field over a considerable distance along the axis. If two identical coaxial coils are placed with their centers separated by a distance equal to their radii, then the total magnetic field is uniform for some distance on both sides of the center point. Two such coils are known as *Helmholtz coils* and are often used to produce a uniform magnetic field.

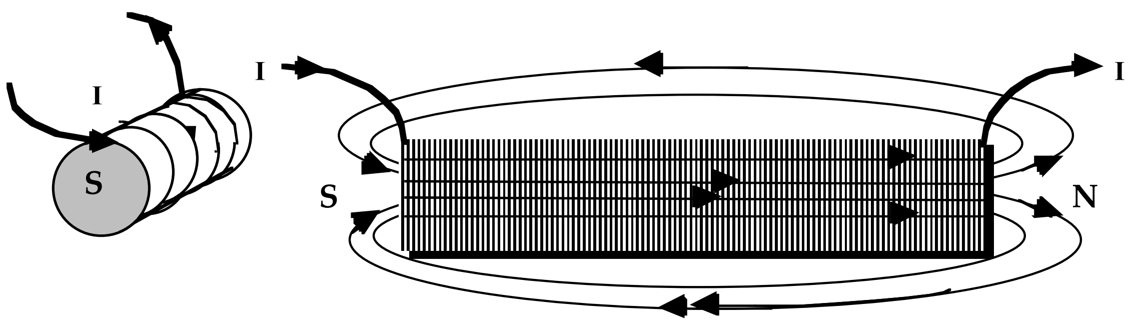
The geometry of *Helmholtz* coils - the radius of the coils is equal to their separation - provides a highly uniform magnetic field. If each coil has N turns and has radius *a, then* the magnetic field (B) produced by the *Helmholtz* coils is proportional to the current through the coils (I) and given by the following formula

$$B = \frac{8N\mu_0}{a5\sqrt{5}} I = k \cdot I \,, \tag{1}$$

where μ_0 is the permeability constant ($\mu_0 = 4\pi * 10^{-7}$ T·m/A). The magnetic field (B) is measured in teslas (T) and current (I) in amperes (A). The calibration coefficient, $k = B/I$ (T/A), gives the measure of how many teslas we would get from 1A current through the coils.

D. Solenoids

A solenoid (a coil) is a long wire that is wrapped into a tight helical coil of many closely spaced loops. When current flows through the solenoid, a strong magnetic field is created that allows the solenoid to act as a magnet with both a north pole and a south pole.



By reversing the direction of the current flowing through the solenoid, one can swap north and south poles of the solenoid. Inside the **ideal solenoid** (one which is **infinitely long** with very tight coils), the magnetic field lines are parallel to the axis and are equally spaced. In other words, the magnetic field is uniform. It has the same magnitude everywhere within the solenoid. **The magnetic field inside an exceptionally long solenoid is proportional to the current and to the number of turns per unit length of the solenoid:**

$$B = \mu_o n I_o = \mu_o \frac{N}{L} I_o \qquad n = \frac{N}{L} \tag{2}$$

where:

μ_o = a constant = $1.2566\times10^{-6}\,\frac{\mathrm{N}}{A^2} = 1.2566\times10^{-6}\,\frac{T\ m}{A} = 1.2566\times10^{-2}\,\frac{Gs\ m}{A}$

n = the number of turns per unit length of the solenoid = N/L

where: N is the total number of turns, L is the length of the solenoid

I_o = the current running in the solenoid

The slope m_{Eq2} of the magnetic field B vs. current I for infinitely long solenoid (coil) is equal to

$$\text{B vs. I slope } m_{Eq2} = \mu_0 n = \mu_0 \frac{N}{L}$$

For real solenoids, which are **not** infinitely long, but have the length L and the radius R, the Eq. (2) changes into

$$B = \frac{\mu_0 n I_0}{\sqrt{1+\left(2R/L\right)^2}} = \mu_0 n I_0 \frac{1}{\sqrt{1+\left(2R/L\right)^2}} \tag{3}$$

In case of an exceptionally long solenoid, i.e., a solenoid with the length much larger than its radius

$$L >> R \quad \Rightarrow \quad \left(2R/_L\right) \to 0 \;\; ; \;\; \sqrt{1+\left(2R/_L\right)^2} \to \sqrt{1+0} = \sqrt{1} = 1$$

$$B = \frac{\mu_0 n I_0}{\sqrt{1+\left(2R/_L\right)^2}} \to \mu_0 n I_0$$

and Eq. (3) becomes Eq. (2). For real solenoids, Eq. (3) offers a much better approximation than the formula for infinitely long solenoid.

The slope m_{Eq3} of the magnetic field B vs. current I for a solenoid (coil) with length L and radius R is equal to

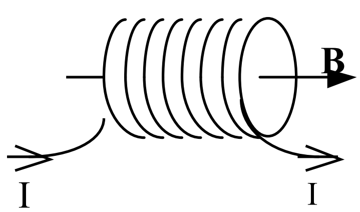
$$B \text{ vs.} I \text{ slope } m_{Eq3} = \frac{\mu_0 n}{\sqrt{1+\left(2R/_L\right)^2}} = \frac{\mu_0 N}{L * \sqrt{1+\left(2R/_L\right)^2}}$$

The magnetic field at the center of a single, circular current loop with $N = 1$ and $L \cong 0$ is

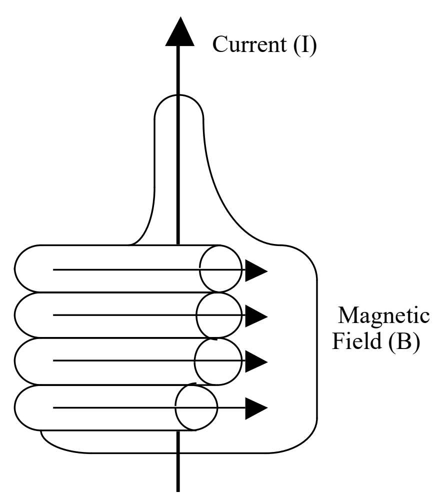
$$B = \frac{\mu_0 N}{L * \sqrt{1+\left(2R/_L\right)^2}} = \frac{\mu_0 N}{\sqrt{L^2+4R^2}} = \frac{\mu_0 I}{2R} \tag{4}$$

where R is the radius of the loop and I is the current in the loop.

The direction of the magnetic field inside the solenoid is governed by the following rule. Use your right hand and let your thumb represent the direction of the current. Curl your other fingers. They indicate the direction of the magnetic field vector $\vec{B}$. Check the two pictures below:



I. Solenoid

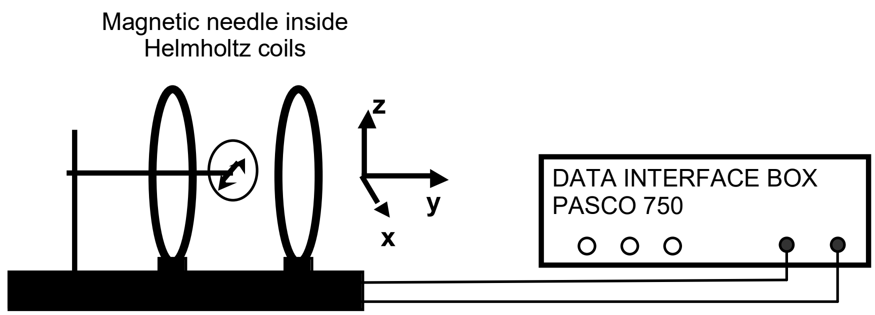


II. Right Hand Rule

Procedure:

Activity 1: Horizontal and Vertical Components of the Earth's Magnetic Field

In this activity, you will measure both components (horizontal and vertical) of the Earth's magnetic field. The experimental setup includes magnetic needle located inside a pair of Helmholtz coils and the PASCO 750 data interface.



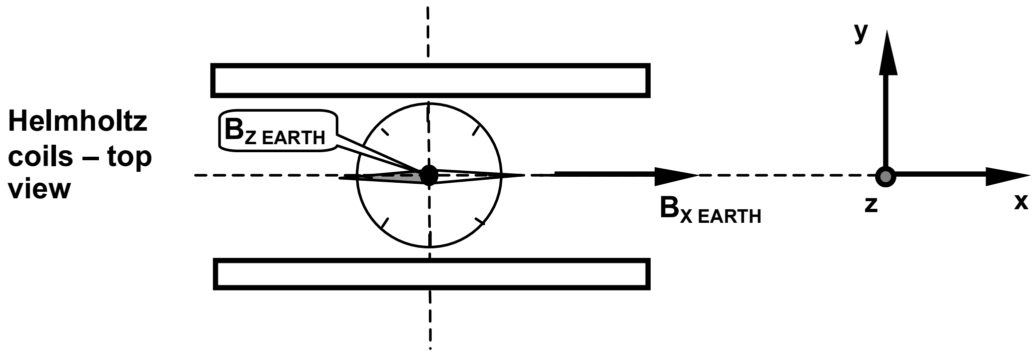
The magnetic needle is attached to a circular, plastic protractor for measuring angles and can freely rotate in the plane of the circular protractor.

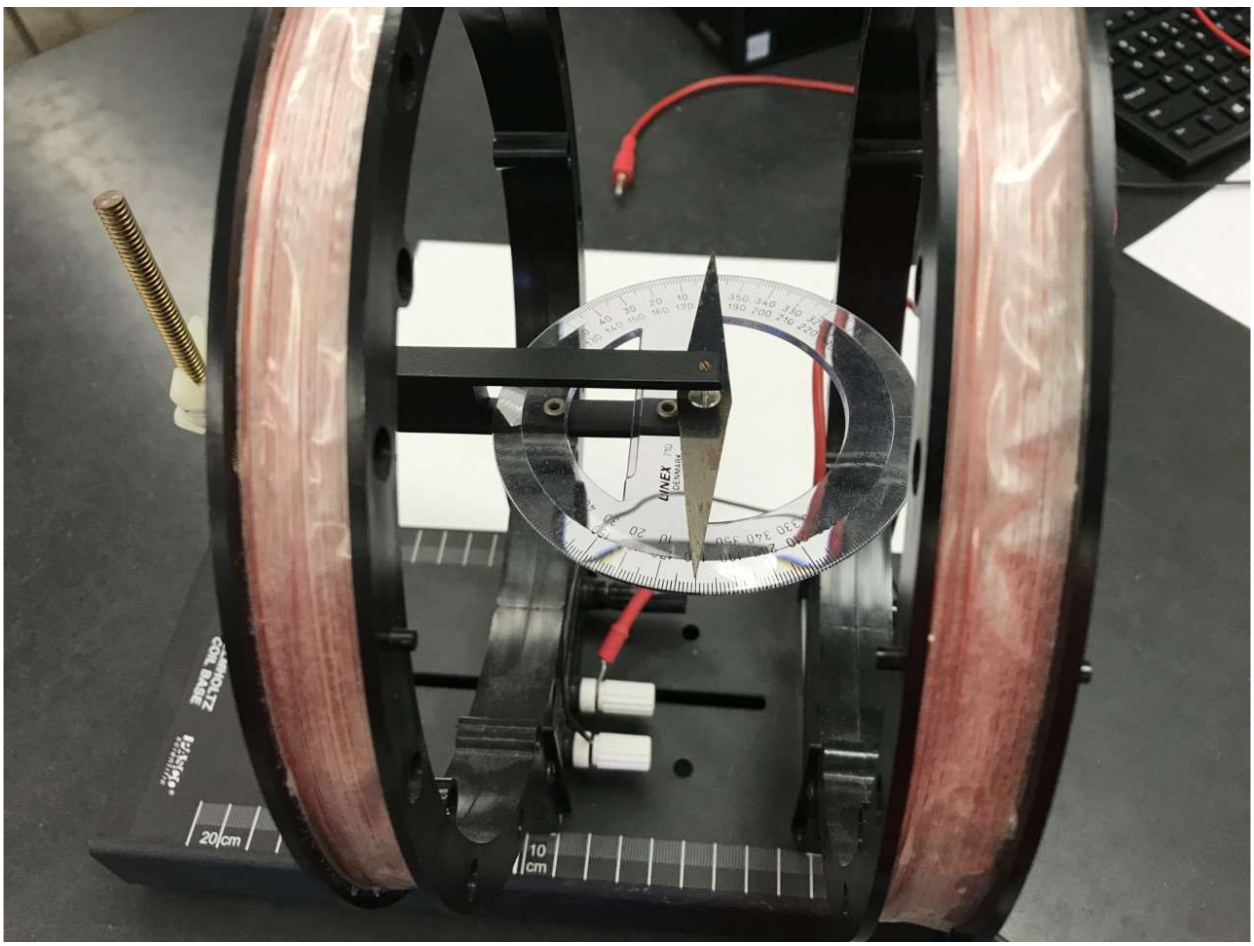
For measuring the horizontal component, the magnetic needle should rotate in the horizontal plane. However, the magnetic needles and the protractor assembly can be rotated along the horizontal axis, so the needle could rotate in vertical plane. We will use that second orientation to measure the vertical component of the Earth's magnetic field.

1.1. First, make sure that the magnetic needle can rotate in the horizontal plane like an ordinary compass.

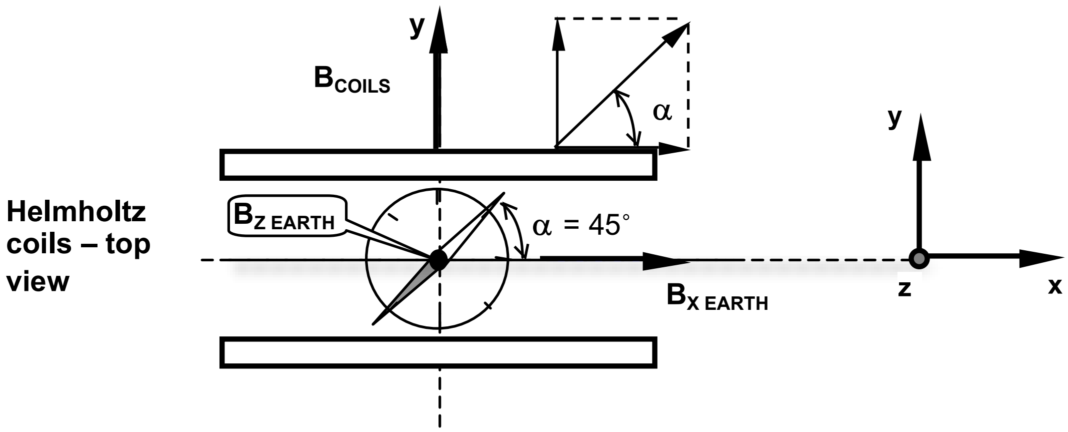
We have selected the following frame of reference: the x-axis is horizontal and goes in between the Helmholtz coils, the y-axis is also horizontal and parallel to the symmetry axis of the circular coils, and z-axis is vertical.

1.2. Rotate the Helmholtz coils with the base (no current yet) so that the Earth's magnetic field vector is in the plane that is parallel to the coils (i.e., in the x-z plane). The magnetic needle should be parallel to the plane of the coils. For this configuration, the Helmholtz coils can produce a magnetic field that is perpendicular to the coils and at the same time perpendicular to the Earth's magnetic field. You should have the following situation without any current in the coils ($I = 0$). **By rotating the Helmholtz coils, you rotate the frame of reference**. The Earth's magnetic field vector should now be in the x-z plane, i.e., no y-component of the Earth's magnetic field.





1.3. The coils produce magnetic field, which is perpendicular to the circular coils, i.e., it is <u>aligned with y-axis</u>. With a gradual increase of the current, the magnetic field produced by the coils would also increase and the needle would gradually rotate towards B_{COILS} (i.e., increase of angle α).



The magnetic field produced by the coils (B_{COILS}) is always along y-axis, whereas the Earth's magnetic field vector is in the x-z plane. These two vectors are perpendicular to each other. The magnetic needle aligns with the net magnetic field, which is the vector sum of these two fields: B_x (earth field) and B_y (coils). Continue increasing the current until the magnetic needle changes its direction from the initial position ($I = 0$). When the angle $\alpha = 45°$, then

$$\alpha = 45° \quad \Rightarrow \quad |B_{X\,\mathrm{EARTH}}| = |B_{\mathrm{COILS}}|$$

By measuring the current necessary to get the rotation equal to 45° and calculating $|B_{COILS}|$, we will be able to measure the horizontal component of the Earth's magnetic field $|B_{X\,EARTH}|$.

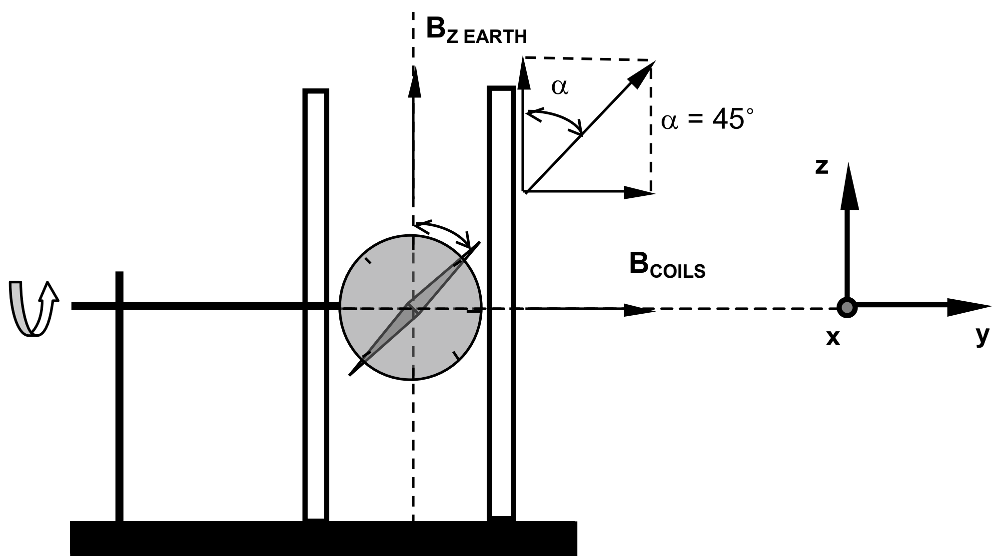
1.4. The magnetic field produced by the Helmholtz coils is proportional to the current. The number of turns in each coil is equal to N = 200 and radius of coils is equal to $a = 0.1025$ m. Using Eq. (1) calculate the calibration coefficient k of the Helmholtz coils and write it in your lab report.

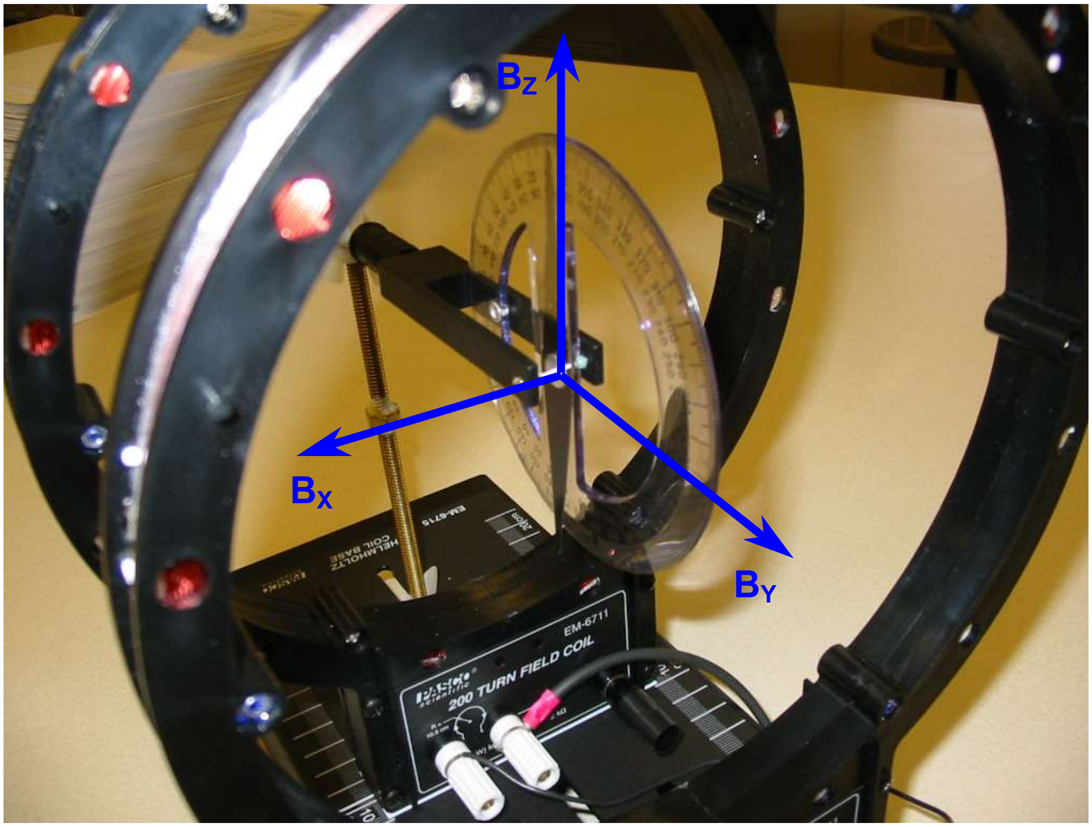
1.5. Make sure that the Helmholtz coils are connected to the "OUTPUT" terminals of the PASCO 750 data interface box.
First, click on the "Record" button. To change the value of the current, adjust the output DC voltage applied to the Helmholtz coils. Use the small up or down arrows or type the voltage number in the "DC Voltage" box located in the "SW750 Output" control window. The output voltage could be any number from the -4V to +4V range. Make small adjustments to get the desired 45° deflection of the magnetic needle. Record the absolute value of the current on the data sheets.

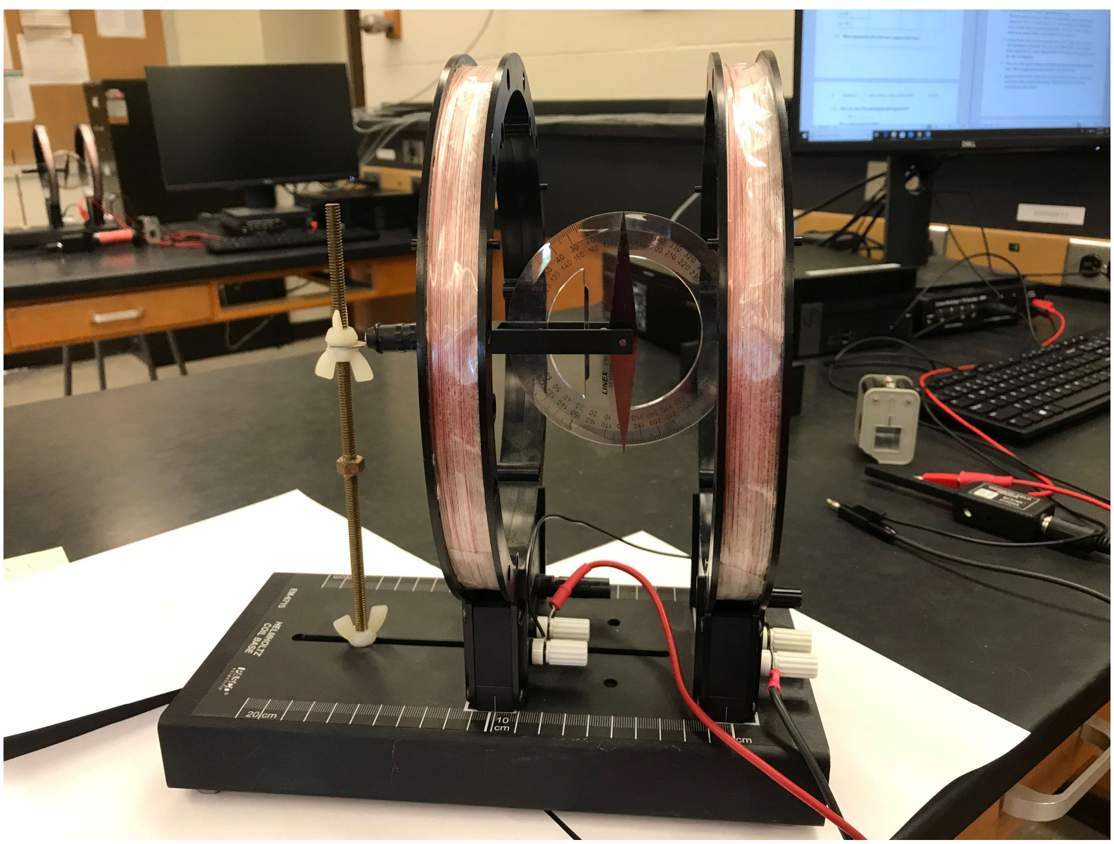
1.6. Reduce the current to zero, then reverse the direction of current in the Helmholtz coils by selecting a negative current. Gradually adjust the current by clicking on up or down arrows until the needle is pointing at α = -45° (or α = 360°- 45°= 315°) angle on the protractor's scale. Record the absolute value of the current on the data sheets.

1.7. Calculate the average value of current in the Helmholtz coils for the 45° deflection of the magnetic needle. Using Eq. (1) calculate the average $|B_{COILS}|$ and then calculate the value of $|B_{X\ EARTH}|$.

1.8. Rotate the assembly of the magnetic needle and the protractor, so the needle can move in the vertical plane (see the figure and the photo below). Keep the same orientation of the Helmholtz coils with respect to the Earth's magnetic field (do not move the coils).







The **vertical component** of the Earth's magnetic field $B_{Z\ EARTH}$ does not change, but we can change the B_{COILS} by changing the current in the coils. With an increase of the current in the coils, the magnetic field produced by the coils would also increase and the needle would gradually rotate towards B_{COILS} (i.e., increase of angle α). If the angle $\alpha = 45°$, then

$$\alpha = 45° \quad \Rightarrow \quad |B_{\mathrm{Z\ EARTH}}| = |B_{\mathrm{COILS}}|$$

By measuring the current necessary to get the rotation equal to 45° and calculating B_{COILS}, we will be able to measure the vertical component of the Earth's magnetic field $|B_{Z\ EARTH}|$. The magnetic field produced by the coils (B_{COILS}) is always along y-axis, whereas the Earth's magnetic field vector is in the x-z plane. These two vectors are perpendicular to each other.

For this orientation and without the current in the coils, the magnetic needle should be vertical. Unfortunately, some of the magnetic needles are not carefully balanced and will not stay vertical. If the effect is not too large, then reversing current should average out these zero offsets.

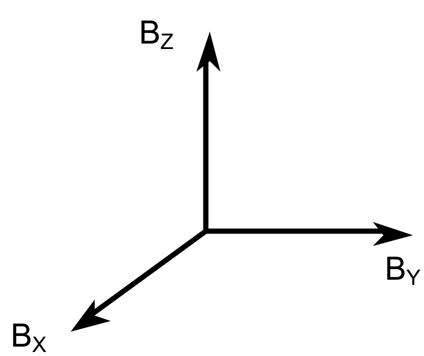
Use the same method of adjusting current as for the measurement of the Earth's magnetic field horizontal component. Record the absolute value of the current on the data sheets.

1.9. Reduce the current to zero, then reverse the direction of current in the Helmholtz coils by selecting a <u>negative</u> current. <u>Gradually</u> increase the current until the needle is pointing at $\alpha = -45°$. The needle should rotate in the opposite direction now. Record the absolute value of the current on the data sheets.

1.10. Calculate the <u>average</u> value of current in the Helmholtz coils for the 45° deflection of the magnetic needle. Then, calculate the average $|B_{COILS}|$ and therefore the $|B_{Z\ EARTH}|$.

1.11. Which component B_X or B_Z of the Earth's magnetic field is larger?

1.12. The magnetic field vector has three components: B_X, B_Y, and B_Z. You have already measured B_X and B_Z. What is the value of the third magnetic field component B_Y? You do not need to make any measurements to answer this question!



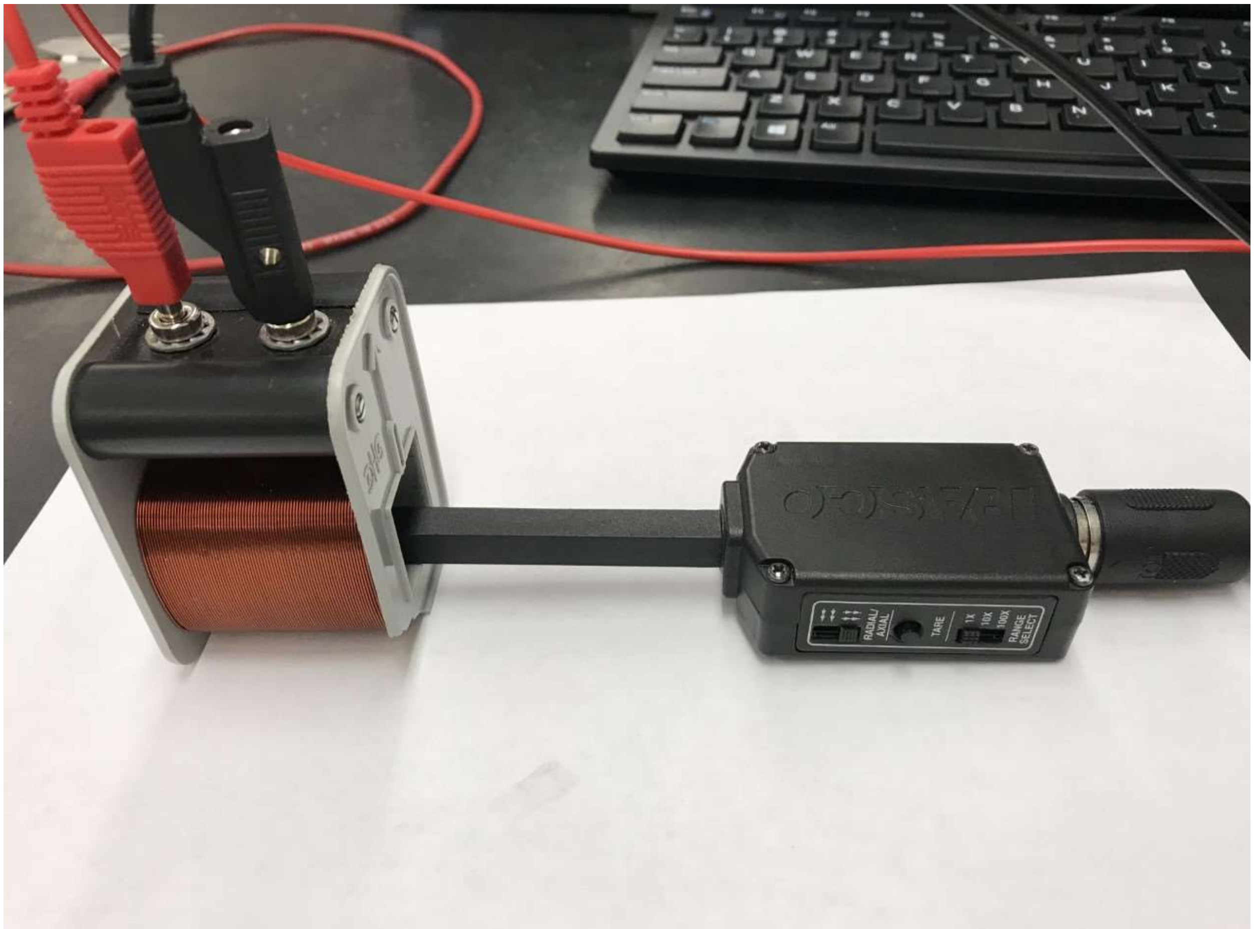
1.13. Calculate the absolute value of the Earth's magnetic field $|B_{EARTH}|$.

1.14. Calculate the angle between the Earth's magnetic field vector and the horizontal (the dip angle).

Activity 2: Magnetic Fields Produced by Coils

In this activity, you will measure the magnetic field produced by a current running through a coil. First, disconnect the Helmholtz coils from the data interface. The coils that we will be using (shown below) may be considered as non-ideal solenoids. Locate the 800-turn coil and connect it to the data interface as shown below.

2.1. To measure the magnetic field of the coil as current runs through it, you must first place the tip of the sensor inside the coil as shown below. <u>Set</u> the **"RANGE SELECT"** slide on the magnetic field sensor to "**1**" and set the **"axial"** mode.



2.2. Locate the file "E5, Activity 2" and double click to open it.
The data recording process goes as follows. The data interface supplies slowly changing voltage from +2.8V to –2.8V and then back to +2.8V. At the same time, the Signal Interface keeps recording values of current and magnetic field. After 20 sec of data recording, you should see the magnetic field vs. current graph for the investigated coil.

2.3. Confirm that the sensor is set to the axial mode and press the "TARE" button (before you start collecting data in this mode). Then, click on the "**Start**" button. A plot will appear for the magnetic field vs. current. What is the value of the slope (m) of this graph? Record this value on the data sheet.

2.4. Print a copy of this graph by making it active display and then selecting **Print** from the **File** menu. Write your name on the printout and label it: "E5, 800-turn coil".

2.5. Reverse the direction of the current in the coil (set the voltage to the negative value or swap red and black cables) and repeat measurements. What is the sign of the slope for the reversed direction of the current?

2.6. Calculate the average absolute value of the slope $|m|$ for the 800-turn coil.

2.7. Using the two formulas for the magnetic field inside a solenoid **calculate the theoretical values of the B vs. I slope** for the 800-turn coil. The first formula is for the infinitely long solenoid (Eq. (2)), whereas the more accurate formula valid for a finite length solenoid is Eq. (3). Both formulas predict that the magnetic field inside the coil (B) is proportional to the current in the coil, but they predict different coefficient of proportionality, which is the slope of the B vs. I graph. The average radius of both these coils is equal to $R = 0.018$ m. The 800-turn coil is not cylindrical, so even Equation (3) for a finite length coil does not describe the magnetic field exactly!

2.8. Find the number of turns for the "unknown", red coil.

Replace the 800-turn coil with the "unknown", red coil. This coil has the length $L = 0.110$ m and the average radius of the coil is equal to $R = 0.0175$ m. Measure the magnetic field vs. current $B(I)$ to find the number of turns N for that coil. How big is the finite length solenoid correction factor for the "unknown", red solenoid?

2.9. Disconnect the "unknown", red coil and re-connect the Helmholtz coils to the "OUTPUT" terminals of the PASCO 750 data interface box.

Make sure to complete the following tasks:

You must submit the answers to the prelaboratory questions online. (3.5 points)

1. One graph from *Activity 2*. (1 point)
   (Title and write your name and those of your partners on each graph.)
2. Your completed Data Sheets. (5.5 points)
3. Return the completed lab report to your lab TA.